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Safe Longitudinal Platoons of Vehicles without Communication

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Abstract: This report deals with the platooning problem that can be defined as the automatic following of a manned driven vehicle by a convoy of automatic ones. Different approaches have been proposed so far. Some require the localisation of each vehicle and a communication infrastructure, others called near-to-near approach only needs vehicle on-board sensors. However, to our knowledge, they do not provide any proof of non collision. We propose a novel near-to-near longitudinal platooning building a collision-free platooning whatever the number of vehicles. The model is derived from the study of the most dangerous interaction between two vehicles, *i.e.* considering the maximum acceptable acceleration when the previous vehicles brakes at maximum capacity. Collision avoidance of this model is proved. Finally, we show that this model can be combined to existing ones, keeping this collision-free property while allowing more various behaviors.

Key-words: Intelligent Vehicle, Platooning, Decentralized Approach, Multi-Agents System, Collision Avoidance, Proof

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Contrôle longitudinal sans collision pour un convoi de véhicules sans communication

Résumé : Ce rapport se consacre au problème de la conduite de véhicules en convoi : il s'agit de définir un comportement permettant à des véhicules de suivre en automatique un premier véhicule conduit par un chauffeur. Différentes approches ont été proposées jusque là. Certaines requièrent la localisation de chaque véhicule et une infrastructure de communication, d'autres dites de proche en proche n'ont besoin que de capteurs embarqués. Cependant, aucune ne fournit à notre connaissance de preuve de non-collision. Nous proposons une nouvelle approche de proche en proche pour assurer un contrôle longitudinal sans collision, quel que soit le nombre de véhicules du convoi. Ce contrôle découle de l'étude de l'interaction la plus risquée entre deux véhicules consécutifs : on considère la plus grande accélération évitant la collision lorsque le précédent véhicule freine au maximum de ses possibilités. On prouve alors que ce contrôle permet d'éviter toute collision. Enfin, on montre comment ce contrôle peut être couplé à d'autres méthodes, préservant cette propriété de non-collision tout en permettant des comportements plus variés.

Mots-clés : Véhicule intelligent, convoi, approche décentralisée, système multi-agent, évitement de collision, preuve

Contents

1 Introduction

Future urban transportation systems will include autonomous vehicles. The public transportation system we envision would be made up of a fleet of small electrical vehicles (often called CyberCar, *cf.* Fig. 1) specifically designed for areas where car traffic must be severely restricted. This system could cover the urban areas with low demands or outside peak hours.

Figure 1: CyberCar platoon with Cycabs (Mobivip exhibition, Nancy 2005).

This new mode of public transport needs to be effective to ensure that there is always a balance between supplied and requested vehicles. A solution is to collect the vehicles distributed in areas with lower request. We imagine that a pilot could drive a leader vehicle, collecting vehicles that would automatically follow it to form a train without grip material.

In this paper we focus on the longitudinal platoon problem, by considering that vehicles are moving in a one dimensional space (along a line). As it has been shown that longitudinal and lateral control can be dealt separately [1], results obtained in this paper can be extended to the general platooning problem.

We consider the near-to-near approach as it does not require any infrastructure (GPS, wireless communication, etc.) which introduces additional complexity and unreliability in a platooning system. We think that such an approach is better suited to deal with the open challenge of ensuring that no collision can occur within platoons.

Several approaches have been explored to deal with near-to-near longitudinal platooning. Among the most typical models, we can quote the following references. Sheikholeslam and Desoer [4] proposed a longitudinal control based on linearization methods that focuses on stability of the convoy. Platoon stability is also tackled in [3] where a fixed-gain PID control with gain scheduling is used. By contrast, a control mode based on a non-linear method with PID is proposed in [2], dealing with train stability but where collision accidents are assumed to be possible. As it was designed for CyberCars, we particularly examine in this paper the model introduced by Daviet and Parent [1] which relies on linear corrector with variable coefficients. Contrarily to these approaches we tackle the platooning problem by considering collision avoidance as the main criteria to design safe platoons. We propose a novel approach building a collision-free platoon whatever the number of agents.

The model is derived from the study of the most dangerous interaction between two vehicles, *i.e.* considering the maximum acceptable acceleration when the previous vehicles brakes at maximum capacity.

We do not study a particular model of autonomous vehicle, but we consider a generic vehicle/robot that may be controlled through its acceleration set point. We assume that each vehicle/robot owns a low level controller allowing to reach the acceleration set point. In order to make this assumption realistic we bound acceleration and speed values, and we consider a time delay to reach the set point.

The paper is organized as follows. Section 2 defines the longitudinal platooning problem. Then, in section 4, we illustrate on the classical Daviet & Parent approach that such a model do not ensure avoiding collisions. In section 5 we propose a collision-free platooning model, prove this property and show how to combine this approach with other platooning models. Section 6 discusses the robustness to perception uncertainty. Finally section 7 concludes.

2 The Platooning Problem

2.1 Hypotheses

We consider a set of N ($\geqslant 2$) vehicles forming a linear platoon. The first vehicle, numbered 0, is driven by a human being. The others, numbered from 1 (following the leader) to $N-1$, are controlled by autonomous agents (*cf.* Fig. 2).

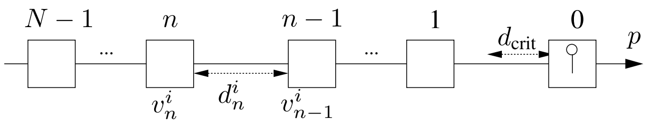

Figure 2: Scheme of the problem and notations.

Motion of the considered vehicles is limited by fixed bounds on their velocity and acceleration, respectively called v_{min}, v_{max}, a_{min} and a_{max}. We suppose that:

$$0 \leqslant v_{\text{min}} < v_{\text{max}} \text{ and } a_{\text{min}} < 0 < a_{\text{max}}$$

We consider forward-only motions, with accelerations and decelerations. These limits can be due to traffic laws, passengers' comfort or any other constraints. They are generally far from engine's limits, to be sure that the low-level controller can always achieve the desired acceleration.

Leader's behavior is one of the problem input that greatly influence the platoon behavior. In this paper we simulate strong variations of velocity at different time steps, as presented in section 3.2.

All the autonomous vehicles have a cyclic behaviour:

1. they perceive their environment,
2. they compute an acceleration set point,
3. they send this acceleration to the low-level controller.

We suppose each step is performed simultaneously by all the agents, *i.e.* they are synchronized.

We denote δt the cycle duration, and τ the fixed delay (not known by agents) between perceptions and resulting actions. This delay includes durations for the perception, the acceleration computation and the transmission to low-level. We suppose $\tau < \delta t$.

At instant $i\,\delta t$ ($i \geqslant 0$), vehicle numbered n ($0 < n < N$) perceives:

- its velocity v_n^i,
- its distance to previous (numbered $n-1$) vehicle d_n^i,
- the previous vehicle's velocity v_{n-1}^i.

Note that positions p_n^i are not known by the vehicles. Contrary to infrastructure-based approaches we do not need to localise vehicles.

Actual sensors can estimate distances with such an accuracy that velocity of previous vehicle can be derived from it (using vehicle velocity). According to these perceptions, each vehicle decides which acceleration a_n^i to apply to itself from instant $i\,\delta t + \tau$ to $(i+1)\,\delta t + \tau$.

Note: We present in section 6 how errors in perceptions can be considered in the model.

2.2 Non-Collision Property

Collision avoidance is usually formulated as a **strict** inequality: distances should remain strictly positive. This induces that, at each time step, chosen accelerations should respect **strict** inequalities (to avoid future collisions): there is no optimal solution, as acceleration can be chosen as close as wanted to the forbidden limit.

We choose to define a critical distance $d_{\text{crit}} > 0$, which may be as small as wanted, and consider that collision is avoided when distances between vehicles are *greater than or equal to* d_{crit}: $d_n^i \geq d_{\text{crit}}$. Non-collision constraint for accelerations thus admits an optimal (maximal) solution, which is used for our method (in section 5.1).

3 Experimental Framework

3.1 Simulation Model

In this paper, experiments are performed in simulation. The acceleration a_n^i changes the position and velocity of the vehicle numbered n, according to elementary dynamic laws (straight motion with constant acceleration). However, bounds on velocities and accelerations make formulas more complex. The function $\texttt{move} : (p, v, a, t) \mapsto (p, v)$ computes the couple (position, velocity) obtained when acceleration a is applied for a duration t, starting from position p with a velocity v.

$$\texttt{move}(p, v, a, t) = \begin{cases} (v_{\min}, p + v_{\min}\, t - (v - v_{\min})^2/a) & \text{if } (v + a\,t < v_{\min}) \\ (v_{\max}, p + v_{\max}\, t - (v_{\max} - v)^2/a) & \text{if } (v + a\,t > v_{\max}) \\ (v + a\,t, p + v\,t + a\,t^2/2) & \text{otherwise} \end{cases}$$

Moreover, as acceleration is only applied after a delay τ, the couple (position, velocity) is computed in two steps. First, values are computed at instant $i\,\delta t + \tau$: $(p', v') = \texttt{move}(p_n^i, v_n^i, a_n^{i-1}, \tau)$, *i.e.* the previous acceleration a_n^{i-1} is used before the delay ($a_n^{-1} = 0$). Then, new position and velocity are computed at instant $(i+1)\,\delta t$: $(p_n^{i+1}, v_n^{i+1}) = \texttt{move}(p', v', a_n^i, \delta t - \tau)$.

3.2 Experiments' Characterization

As we consider several experiments in the following, the problem's data corresponding to each experiment will be given in the form of a vector, called a *configuration*. This vector contains v_{min}, v_{max}, a_{min}, a_{max}, the initial distances d^0 and velocities v^0, a minimum aimed distance Δ (used in most of the controllers), δt, τ and the leader's behavior. The minimum distance d_{crit} and the number N of vehicles are not included in the configuration, as they are constant for the experiments presented in this article: d_{crit} is set to 0.05 meter, and 6 vehicles are considered (experiments with up to 12 vehicles gave similar results, with graphics harder to read — color graphics[1] for any number of vehicles can be obtained with the on-line simulator, available at `http://www.loria.fr/~scheuer/Platoon`).

The leader's behavior is given as a sequence of k ($k > 0$) couples, denoted $(t_i, v_0^i), 0 \leqslant i < k$. We suppose that the couples are ordered by increasing values of time, *i.e.* that $t_{i-1} < t_i$ ($1 \leqslant i < k$), with $t_0 = 0$. Considering that $t_k = +\infty$, time axis can be cut in k intervals $I_i = [t_i, t_{i+1}[$.

Each couple indicates the velocity the leader tries to reach, after the associated instant. At instant t_i (leader reaction is not delayed by τ), the leader acceleration a_0^i is set to the optimal acceleration to reach velocity v_0^i:

$$a_0^i = a_{\text{min}} \text{ if } v_0^i < v_0(t_i) \qquad \text{and } a_0^i = a_{\text{max}} \text{ otherwise}$$

Acceleration is maintained until v reaches the desired velocity v_0^i or t reaches t_{i+1}, the end of interval I_i. Acceleration thus stops at $t_i^* = \min(t_{i+1}, t_i + (v_0^i - v_0(t_i))/a_0^i)$. Note that velocity v_0^i may not be reached, if $|v_0^i - v_0(t_i)|$ is too big with respect to $|t_{i+1} - t_i|$ and either a_{min} or a_{max}.

The velocity profile of the leader is thus a piecewise linear curve as illustrated in Fig. 3.

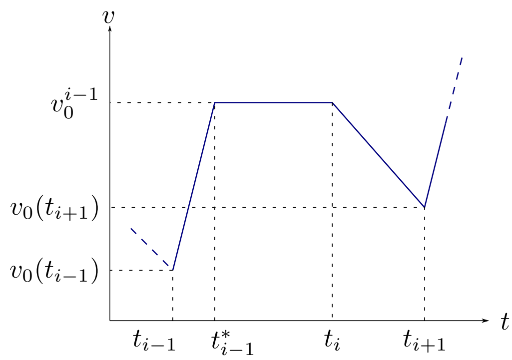

Figure 3: Velocity profile for the leader.

[1] Experimental results are presented here as color graphics, color changing from blue for the vehicle #1 to green for #5. Greyscale print of this report will make these graphics harder to understand. In that case, color graphics can easily be obtained using the on-line simulator.

4 Study of Daviet and Parent Models

Daviet and Parent [1] present a controller which compute acceleration a_n^i as a function of d_n^i, v_n^i and v_{n-1}^i, using the general formula:

$$a_n^i = \frac{(d_n^i - \Delta - hv_n^i)/C_d + v_{n-1}^i - v_n^i}{C_v},$$

where Δ is the minimum aimed distance and h is a "reaction delay" experimentally fixed to 0.35 s by authors. Two variants of the controller are proposed:

- constant coefficients controller uses $C_d = C_v = h$,
- variable coefficients controller uses $C_v = h$ and $C_d = \max(h, v_n^i\ /a_{\text{max}})$.

Various experiments of these controllers has shown four global behaviors:

B1. accelerations and velocities quickly reach the intended values (without oscillation or long evolution);

B2. if the value Δ does not respect some constraints (typically, it has to be higher than a certain value), oscillations (and sometimes collisions) appear;

B3. the real distances between vehicles may be quite smaller than the distances aimed at each cycle ($\Delta + hv$), collisions may thus appear;

B4. the constant coefficients controller seems less stable than the variable coefficients one; oscillations are more frequently observed, and differences between real and aimed distances are greater.

Behavior B1 of the constant coefficients controller is illustrated in Fig. 4, with $a_{\text{max}} = -a_{\text{min}} = 2$ m/s^2, $v_{\text{min}} = 0$ m/s and $v_{\text{max}} = 14$ m/s (50.4 km/h), d^0 = 3 m and v^0 = 0 m/s, $\delta t = 0.01$ s and $\tau = 0.007$ s, $\Delta = 0.15$ m, and the following behavior for the leader:

t (s)	0	8	16	24	32
v_0 (m/s)	14	0	14	0	10

Behavior B2 of the constant coefficients controller is illustrated in Fig. 5 with $a_{\text{max}} = -a_{\text{min}} = 0.5$ m/s^2, $v_{\text{min}} = 0$ m/s and $v_{\text{max}} = 8$ m/s (28.8 km/h), d^0 = 3 m and v^0 = 0 m/s, $\delta t = 0.01$ s and $\tau = 0.007$ s, $\Delta = 0.17$ m, and the following behavior for the leader:

t (s)	0	17.5	35	52.5	70
v (m/s)	8	0	8	0	6

Setting $\Delta \geqslant 0.18$ m seems necessary to switch to a normal behavior, *i.e.* without collision.

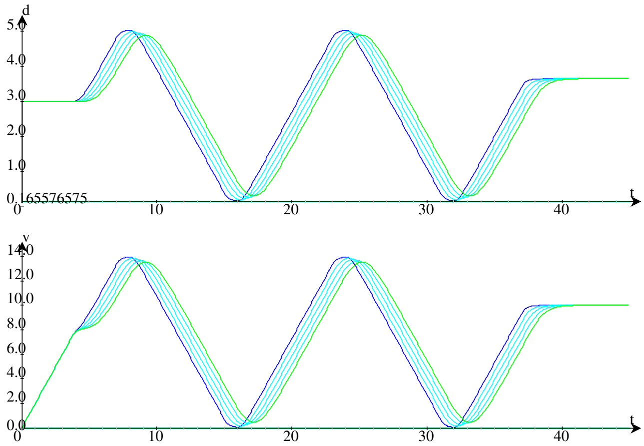

Figure 4: Standard result using D & P controllers.

Behavior B3 of the variable coefficients controller is illustrated in Fig. 6, with $a_{\min} = -1$ m/s^2, $a_{\max} = 2$ m/s^2, $v_{\min} = 0$ m/s and $v_{\max} = 14$ m/s, $d^0 = 3$ m and $v^0 = 0$ m/s, $\delta t = 0.01$ s and $\tau = 0.007$ s, $\Delta = 0.2$ m, and the following behavior for the leader:

t (s)	0	7.5	22
v (m/s)	14	0	10

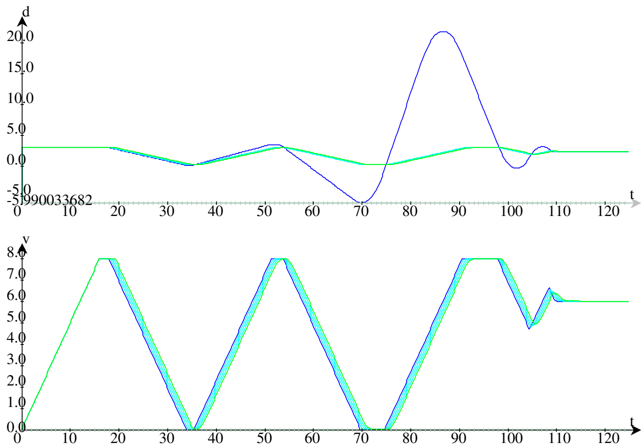

Figure 5: D & P constant coefficients controller in collision.

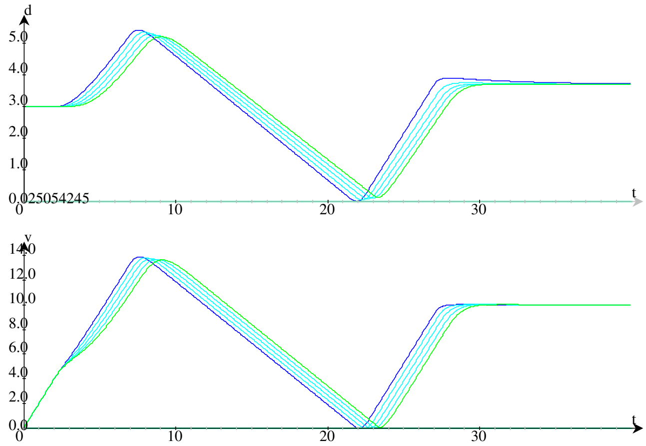

Figure 6: D & P variable coefficients controller in collision.

In Fig. 6, minimum distance between the vehicles comes as low as 0.025 m, which is much less than Δ (0.2 m) and even less than d_{crit} (0.05 m): collision occurs!

Oscillations (and often collisions) also appear when initial distances d^0 is smaller than the dynamic ideal distance associated to initial velocity v^0, *i.e.* when $d^0 < \Delta + hv^0$. This constraint on initial values can be quite restrictive, due to the high value of h (0.35 s).

To avoid this constraint's effect, dynamic ideal distance can be reduced by changing the value of h to a multiple of the time step δt instead of using 0.35 s. This however reduces the stability of controllers: oscillations appear more frequently. We will see in section 5.2 how this can be avoided.

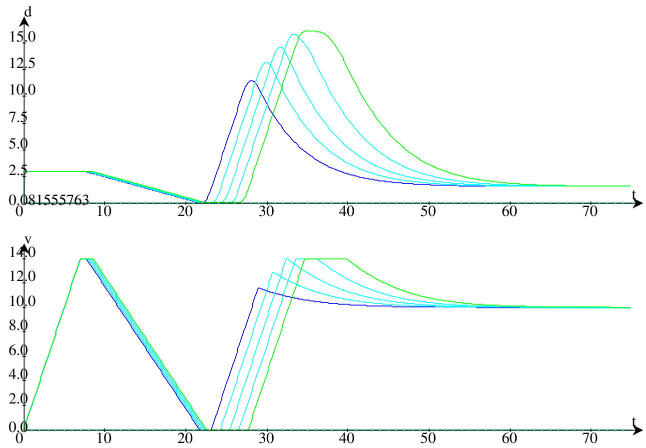

Figure 7: Fast D & P controller.

To illustrate this behavior, we define a variant of the variable coefficients controller with $h = 2\delta t$. We call it *fast D & P*. Fig. 7 shows its behavior with the same configuration as for Fig. 6, except Δ which is much higher (1.4 m instead of 0.2 m). Difference between Δ and minimum distance obtained is now 0.6 m (1.4 - 0.8), instead of only 0.175 (0.2 - 0.025). Moreover, the velocity do not reach efficiently the final aimed velocity: it takes more than 45 s to stabilize, while original D & P controllers usually need less than 10 s.

Finally, there is no analytical method to find the minimum aimed distance Δ so as to avoid collision for an infinite set of configurations (e.g., fixing some of the problem's data — for example v_{min}, v_{max}, a_{min}, a_{max}, δt and τ— and allowing any possible values for the others — d^0, v^0 and the leader's behavior).

5 A Collision-Free Platooning

We want to define a controller which ensures a safe behavior: collision with the others vehicles[2] must always be avoided.

At first, we propose a controller verifying this property. We then show how it can be combined with other controllers in order to obtain different aimed distances, without loosing this property.

5.1 Building a Controller Avoiding Collisions

We want to ensure that vehicle n can avoid collision with vehicle $n-1$ after time $i\,\delta t$, whatever the behavior of the previous one. Thus, we consider vehicle $n-1$ brakes at maximum capacity. A collision-free behavior exists if and only if maximum braking of vehicle n allows to avoid collision. This is true when:

$$d_n^i \geqslant d_{\text{crit}} + \max\left(0, \frac{{v_n^i}^2 - {v_{n-1}^i}^2}{-2a_{\text{min}}}\right) \tag{1}$$

where a_{min} is the minimum acceleration (or maximum deceleration) for all vehicles. Justification of this relation is presented in appendix I. Let $\delta d_n^i = d_n^i - d_{\text{crit}} + \min\left(0, \left({v_n^i}^2 - {v_{n-1}^i}^2\right)/(2a_{\text{min}})\right)$. Inequality (1) can then be written simply $\delta d_n^i \geqslant 0$.

[2]Other obstacles, like pedestrians, are not taken into account. We will see later that this is barely a problem, as distances between the vehicles are too small for an obstacle to interfere.

Analytical developments show this inequality (1) is verified when acceleration a_n^i remains lower than or equal to

$$\begin{aligned} & a_{\text{lim}}(d_n^i, v_n^i, v_{n-1}^i) \\ & = \min\Bigg(a_{\text{min}} + 2\frac{\tilde{d}_n^i - d_{\text{crit}} + (\tilde{v}_{n-1,n}^i - \tilde{v}_n^i)\delta t}{3\delta t^2}, \\ & \qquad \frac{\sqrt{\left(\tilde{v}_n^i - a_{\text{min}}\frac{\delta t}{2}\right)^2 - 2a_{\text{min}}\widetilde{\delta d}_n^i} - \left(\tilde{v}_n^i - \frac{3}{2}a_{\text{min}}\delta t\right)}{\delta t} \\ & \qquad \frac{\sqrt{\left(\tilde{v}_n^i + \left(a_{\text{max}} - \frac{a_{\text{min}}}{2}\right)\delta t\right)^2 - 2a_{\text{min}}\widetilde{D}_n^i} - \left(\tilde{v}_n^i + \left(a_{\text{max}} - \frac{3}{2}a_{\text{min}}\right)\delta t\right)}{\delta t} \Bigg), \end{aligned} \tag{2}$$

where $\tilde{d}_n^i = d_n^i + (v_{n-1}^i - v_n^i)\delta t + (a_{\text{min}} - a_{\text{max}})\delta t^2/2$, $\tilde{v}_{n-1,n}^i = v_{n-1}^i + a_{\text{min}}\delta t$, $\tilde{v}_n^i = v_n^i + a_{\text{max}}\delta t$, $\widetilde{\delta d}_n^i = \tilde{d}_n^i - d_{\text{crit}} + \left(\tilde{v}_n^{i\,2} - \tilde{v}_{n-1,n}^{i\;2}\right)/(2a_{\text{min}})$ and $\widetilde{D}_n^i = \max\Big(0, \widetilde{\delta d}_n^i - (a_{\text{max}} - a_{\text{min}})(\tilde{v}_n^i + a_{\text{max}}\delta t /2)\delta t/(-a_{\text{min}})\Big) + (a_{\text{max}} - a_{\text{min}})\delta t^2$. In this formula, $\tilde{d}_n^i$ is a lower bound of d_n^{i+1}, $\tilde{v}_{n-1,n}^i$ a lower bound of v_{n-1}^{i+1}, $\tilde{v}_n^i$ an upper bound of v_n^{i+1}, $\widetilde{\delta d}_n^i$ a lower bound of δd_n^{i+1}, using the three previous ones, and $\widetilde{D}_n^i$ a lower bound of $\widetilde{\delta d}_n^{i+1}$.

Theorem: With initial constraint $\widetilde{\delta d}_n^0 \geqslant v_n^0\delta t$ $(\forall n, 0 \leqslant n < N)$[3], if $a_n^i \leqslant a_{\text{lim}}(d_n^i, v_n^i, v_{n-1}^i)$ $(\forall n, 0 < n < N$ and $\forall i \geqslant 0)$, then collision cannot occur.

The proof of this theorem are presented in appendix II.

Note: Initial constraint is verified when vehicles are initially stopped, provided that initial distances are large enough. Experimental values remains close to d_{crit}.

The function a_{lim} we just defined only provides an upper bound for the acceleration, in order to avoid collision. To fully define a controller, we still have to select which acceleration to take in the interval $[a_{\text{min}}, a_{\text{lim}}]$. This can be obtained, for example, by taking the minimum value between $a_{\text{lim}}(d_n^i, v_n^i, v_{n-1}^i)$ and a_{max}: we call the resulting controller *closest*, as it tends to minimize distance between the vehicles.

Figure 8 gives an example of this controller's behavior, in the configuration of Fig. 4. Distance is maintained to the minimum possible, except in the beginning where motion of the leading vehicle is the fastest possible (it is thus impossible to reduce the distance); otherwise, when the vehicle moves, the distance is slightly more than d_{crit}, due to reaction time, but remains less than 0.5 m.

[3] This means that vehicles' initial distances are sufficiently safe to insure $\widetilde{\delta d}_n^1 \geqslant 0$.

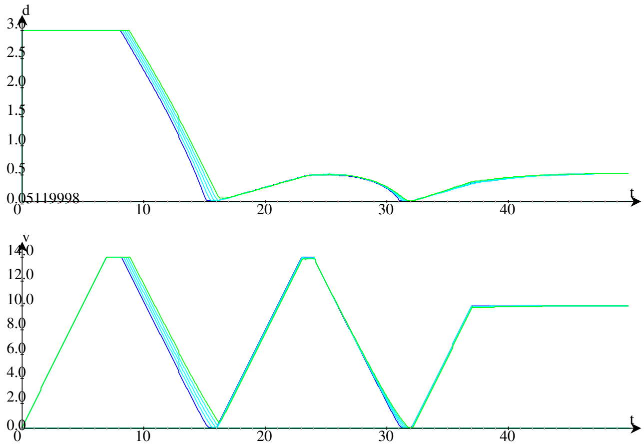

Figure 8: Closest controller in motion.

5.2 Combining with Other Controllers

It is possible to have more various behaviors, by combining $a_{\lim}$ function with another acceleration function γ: selected acceleration is simply the minimum value between $a_{\lim}$ and γ. In that case, $a_{\lim}$ guarantees collision avoidance while γ allows another behavior: it is thus possible to tend to a distance higher than the minimum required to avoid collision.

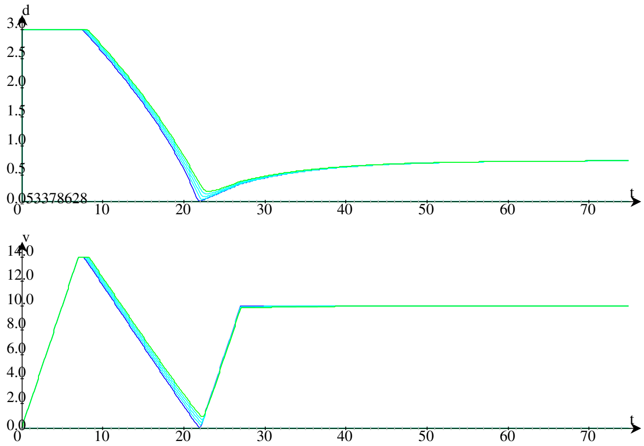

Figure 9: Secure D & P controller avoids collision.

We call *secure D & P* the controller using as γ function the fast D & P controller, as defined at the end of previous section. Fig. 9 shows that this controller avoids collision even with a fixed ideal distance Δ of 0.05 m ($= d_{\text{crit}}$), in the configuration of Figures 6 and 7 where fast D & P controller needed a Δ 28 times higher (1.4 m).

6 Handling Errors in Perceptions

As said at the end of section 2.1, the problem presented in this article does not handle errors in perceptions, for simplicity reasons. These errors can be easily handled, once the algorithm understood, as they are bounded.

Let us call $\varepsilon_d(d)$ the bound on the error in distance perception: if a real distance d is perceived as d^*, $|d - d^*|$ is always lower or equal to $\varepsilon_d(d^*)$. We define similarly ε_v and ε_v^f: ε_v concerns the perception a vehicle has of its own velocity, while ε_v^f concerns the perception of the leading vehicle's velocity.

As we need a lower bound of δd_n^{i+2} to ensure collision avoidance, we need lower bounds of d_n^{i+2} and v_{n-1}^{i+2} and an upper bound of v_n^{i+2}. We then use lower bounds of d_n^i and v_{n-1}^i and an upper bound of v_n^i, computed from their perceived values (resp. ${d_n^i}^*$, ${v_{n-1}^i}^*$ and ${v_n^i}^*$):

$$\begin{cases} d_n^i \geqslant {d_n^i}^* - \varepsilon_d({d_n^i}^*) \\ v_{n-1}^i \geqslant {v_{n-1}^i}^* - \varepsilon_v^f({v_{n-1}^i}^*) \\ v_n^i \leqslant {v_n^i}^* + \varepsilon_v({v_n^i}^*) \end{cases}$$

These formulas can thus be used to replace exact values by perceptions with errors, as long as these errors can be bound (this is generally the case).

7 Conclusion

We investigated in this paper the problem of collision avoidance in near-to-near longitudinal platoons of autonomous vehicles. We first considered the classical Daviet and Parent model, which is one of the rare approach to provide a generic controller separated from the low-level control. A systematic simulation-based analysis of this model has shown that non-collision and stability is not ensured in all cases. We then proposed an alternative model, building a collision-free platoon whatever the number of vehicles. It is derived from the study of the most dangerous interaction between two vehicles, *i.e.* considering the maximum acceptable acceleration when the previous vehicles brakes at maximum capacity. A sketch of the proof of the non-collision property is presented, showing how initial conditions can be fixed. Then, we shown how existing models can be combined to our approach, providing collision avoidance to more various behaviors. At last, these results have been extended to handle uncertainty in perception.

Appendices

These appendix present detailed formulation of every steps of the demonstrations of (1) and of the theorem in section 5.1.

I Security Distance

In this section we justify that (1) ensures that vehicle n can avoid collision with vehicle $n-1$ after time $i\,\delta t$, whatever the behavior of the previous one. We consider the worst case for vehicle $n-1$ (*i.e.* it brakes at maximal rate) and the best case for vehicle n (same behavior): if this does not lead to collision, vehicle n will be able to avoid collision.

As long as none of the vehicles have reached v_{min}, their velocity is a linear function of time, with a constant derivative equal to a_{min}. As a consequence, their velocity difference remains constant. When one of the vehicle reached v_{min}, velocity difference linearly tends to zero as the velocity of the other vehicle linearly decreases toward v_{min}.

Velocity difference thus remains either positive or negative, and distance between vehicles is monotone: its minimal value is obtained either at the beginning or at the end of the motion.

- if $v_{n-1}^i \geqslant v_n^i$, the distance between the vehicles grows or remains constant; collision is then avoided iff $d_n^i \geqslant d_{\text{crit}}$;
- otherwise (if $v_{n-1}^i < v_n^i$), this distance shrinks; times needed to stop are respectively $-v_n^i/a_{\text{min}}$ and $-v_{n-1}^i/a_{\text{min}}$ (let us recall that $a_{\text{min}} < 0$), distances to stop are thus $-{v_n^i}^2/(2a_{\text{min}})$ and $-{v_{n-1}^i}^2/(2a_{\text{min}})$;

  collision is then avoided iff $d_n^i \geqslant d_{\text{crit}} - ({v_n^i}^2 - {v_{n-1}^i}^2)/(2a_{\text{min}})$.

Inequality (1) is a synthesis of these two cases.

Note Inequality (1), rewritten as $\delta d_n^i \geqslant 0$, implies that collision will not occur after one time when both vehicles are braking at maximum rate. In that case, distance is then $d_n^i + (v_{n-1}^i - v_n^i)\delta t$. We thus have, $\forall n, 0 < n < N$ and $\forall i \geqslant 0$:

$$\delta d_n^i \geqslant 0 \Rightarrow d_n^i - d_{\text{crit}} + (v_{n-1}^i - v_n^i)\delta t \geqslant 0$$

As $\widetilde{\delta d}_n^i$ is the equivalent of δd_n^i where $\tilde{d}_n^i$ replaces d_n^i, $\tilde{v}_n^i$ replaces v_n^i and $\tilde{v}_{n-1,n}^i$ replaces v_{n-1}^i, a similar result is obtained $\forall n, 0 < n < N$ and $\forall i \geqslant 0$:

$$\widetilde{\delta d}_n^i \geqslant 0 \Rightarrow \tilde{d}_n^i - d_{\text{crit}} + (\tilde{v}_{n-1,n}^i - \tilde{v}_n^i)\delta t \geqslant 0$$

II Secure Acceleration

This section considers the proof of the theorem presented in section 5.1.

This theorem is proved by recurrence, using initial conditions $\delta d_n^0 \geqslant 0$, $\widetilde{\delta d}_n^0 \geqslant 0$ and $\delta d_n^1 \geqslant 0$, and the implication: $\forall n, 0 < n < N$ and $\forall i \geqslant 0$,

$$\begin{aligned}&\widetilde{\delta d}_n^i \geqslant 0 \text{ and } a_n^i \leqslant a_{\text{lim}}(d_n^i, v_n^i, v_{n-1}^i) \text{ and } a_n^{i-1} \leqslant a_{\text{lim}}(d_n^{i-1}, v_n^{i-1}, v_{n-1}^{i-1}) \text{ if } i > 0 \\ &\quad \Rightarrow \widetilde{\delta d}_n^{i+1} \geqslant 0 \text{ and } \delta d_n^{i+2} \geqslant 0\end{aligned} \tag{3}$$

Recurrence implies that:

- if initial conditions are true:

$$\delta d_n^0 \geqslant 0, \widetilde{\delta d}_n^0 \geqslant 0 \text{ and } \delta d_n^1 \geqslant 0, \forall n, 0 < n < N;$$

- and accelerations are chosen lower than the defined limit:

$$a_n^i \leqslant a_{\text{lim}}(d_n^i, v_n^i, v_{n-1}^i), \forall n, 0 < n < N, \forall i \geqslant 0;$$

- then collision cannot occur:

$$\delta d_n^i \geqslant 0, \forall n, 0 < n < N, \forall i \geqslant 0.$$

In a **first step**, we will prove how initial conditions of the recurrence are obtained using the one of the theorem ($\widetilde{\delta d}_n^0 \geqslant v_n^0 \delta t$): a lower bound of δd_n^1 is computed (in two forms, depending on whether a_n^0 is positive or not), which leads to four conditions containing δd_n^0 and $\tilde{d}_n^0$; $\widetilde{\delta d}_n^0 \geqslant 0$, which is necessary for the recurrence, also insures one of these conditions, for any a_n^0.

More precisely, if $a_n^0 \geqslant 0$, we have

$$v_n^1 \leqslant v_n^0 + a_n^0 \delta t \qquad v_{n-1}^1 \geqslant v_{n-1}^0 + a_{\text{min}} \delta t$$

$$d_n^1 \geqslant d_n^0 + (v_{n-1}^0 - v_n^0)\delta t + (a_{\text{min}} - a_n^0)\frac{\delta t^2}{2}$$

and thus

$$\begin{aligned}\delta d_n^1 \geqslant{}& d_n^0 - d_{\text{crit}} + (v_{n-1}^0 - v_n^0)\delta t + (a_{\text{min}} - a_n^0)\frac{\delta t^2}{2} \\ &+ \min\left(0, \frac{(v_n^0 + a_n^0 \delta t)^2 - (v_{n-1}^0 + a_{\text{min}} \delta t)^2}{2 a_{\text{min}}}\right)\end{aligned}$$

Inequality $\delta d_n^1 \geqslant 0$ is thus implied by

$$\begin{cases} a_n^0 \dfrac{\delta t^2}{2} \leqslant d_n^0 - d_{\text{crit}} + (v_{n-1}^0 - v_n^0)\delta t + a_{\text{min}} \dfrac{\delta t^2}{2} \\ d_n^0 - d_{\text{crit}} + \dfrac{{v_n^0}^2 - {v_{n-1}^0}^2}{2 a_{\text{min}}} + (v_{n-1}^0 - v_n^0)\delta t \\ \quad + (a_{\text{min}} - a_n^0)\dfrac{\delta t^2}{2} + \dfrac{({a_n^0}^2 - {a_{\text{min}}}^2)\delta t^2 + 2(v_n^0 a_n^0 - v_{n-1}^0 a_{\text{min}})\delta t}{2 a_{\text{min}}} \geqslant 0 \end{cases}$$

which can be simplified into

$$\begin{cases} a_n^0 \dfrac{\delta t^2}{2} \leqslant \tilde{d}_n^0 - d_{\text{crit}} + a_{\max}\dfrac{\delta t^2}{2} \\ \delta d_n^0 + (a_n^0 - a_{\min})\left(v_n^0 + a_n^0\dfrac{\delta t}{2}\right)\dfrac{\delta t}{a_{\min}} \geqslant 0 \end{cases}$$

When $a_n^0 \leqslant 0$, the inequality is simplified in a similar manner (a_n^0 is replaced by 0 in the bounds of v_n^1 and d_n^1):

$$\begin{cases} 0 \leqslant \tilde{d}_n^0 - d_{\text{crit}} + a_{\max}\dfrac{\delta t^2}{2} \\ \delta d_n^0 - v_n^0 \delta t \geqslant 0 \end{cases}$$

Let us consider that:

$$\begin{aligned} \widetilde{\delta d}_n^0 &= \tilde{d}_n^0 - d_{\text{crit}} + \frac{\tilde{v}_n^{0\,2} - \tilde{v}_{n-1,n}^{0\;\;2}}{2a_{\min}} \\ &= d_n^0 - d_{\text{crit}} + (v_{n-1}^0 - v_n^0)\delta t + (a_{\min} - a_{\max})\frac{\delta t^2}{2} + \frac{(v_n^0 + a_{\max}\delta t)^2 - (v_{n-1}^0 + a_{\min}\delta t)^2}{2a_{\min}} \\ &= \delta d_n^0 + (v_{n-1}^0 - v_n^0)\delta t + (a_{\min} - a_{\max})\frac{\delta t^2}{2} + \frac{({a_{\max}}^2 - {a_{\min}}^2)\delta t + 2(v_n^0 a_{\max} - v_{n-1}^0 a_{\min})}{2a_{\min}}\delta t \\ &= \delta d_n^0 + (a_{\max} - a_{\min})\delta t \frac{v_n^0 + a_{\max}\frac{\delta t}{2}}{a_{\min}} \end{aligned}$$

Thus, $\delta d_n^0 \geqslant \widetilde{\delta d_n^0}$ and $\tilde{d}_n^0 - d_{\text{crit}} \geqslant \widetilde{\delta d_n^0}$, and the condition $\widetilde{\delta d_n^0} \geqslant v_n^0 \delta t$ implies $\delta d_n^0 \geqslant v_n^0 \delta t$ and $\tilde{d}_n^0 \geqslant d_{\text{crit}}$. As a consequence, first, third and fourth inequalities true.

The remaining inequality can be written $f(a_n^0) \geqslant 0$, where $f(a) = \delta d_n^0 + (a - a_{\min})(v_n^0 + a\delta t/2)\delta t/a_{\min}$ is a quadratic function of negative main factor ($\delta t^2/(2a_{\min})$). It is easy to see that $f(a_{\min}) = \delta d_n^0$ and $f(a_{\max}) = \widetilde{\delta d}_n^0$. As both $f(a_{\min})$ and $f(a_{\max})$ are positive, so is $f(a_n^0)$ for any a_n^0 in the interval $[a_{\min}, a_{\max}]$.

We thus proved that initial conditions of the theorem imply the initial conditions of the recurrence ($\delta d_n^k \geqslant 0$, for $k = 0$ and 1, and $\widetilde{\delta d}_n^0 \geqslant 0$). Together with the recurrence of implication (3), it proves the proposed theorem.

Let us now consider the **proof of implication** (3). We need to find a lower bound of d_n^{i+2}. As this distance depends of velocities v_n and v_{n-1}, evolution of the three are considered:

$$\begin{cases} v_n(i.\delta t + \tau) = v_n^i + a_n^{i-1}\tau \\ v_{n-1}(i.\delta t + \tau) = v_{n-1}^i + a_{n-1}^{i-1}\tau \\ d_n(i.\delta t + \tau) = d_n^i + (v_{n-1}^i - v_n^i)\tau + (a_{n-1}^{i-1} - a_n^{i-1})\dfrac{\tau^2}{2} \end{cases}$$

where $a_n^{-1} = a_{n-1}^{-1} = 0$.

$$\begin{cases} v_n((i+1).\delta t+\tau) = v_n(i.\delta t+\tau) + a_n^i \delta t \\ v_{n-1}((i+1).\delta t+\tau) = v_{n-1}(i.\delta t+\tau) + a_{n-1}^i \delta t \\ d_n((i+1).\delta t+\tau) = d_n(i.\delta t+\tau) + (v_{n-1}(i.\delta t+\tau) - v_n(i.\delta t+\tau))\delta t + (a_{n-1}^i - a_n^i)\dfrac{\delta t^2}{2} \end{cases}$$

This last system can be simplified into:

$$\begin{cases} v_n((i+1).\delta t+\tau) = v_n^i + a_n^{i-1}\tau + a_n^i \delta t \\ v_{n-1}((i+1).\delta t+\tau) = v_{n-1}^i + a_{n-1}^{i-1}\tau + a_{n-1}^i \delta t \\ d_n((i+1).\delta t+\tau) = d_n^i + (v_{n-1}^i - v_n^i)(\tau+\delta t) + (a_{n-1}^{i-1} - a_n^{i-1})\tau(\delta t + \dfrac{\tau}{2}) \\ \qquad + (a_{n-1}^i - a_n^i)\dfrac{\delta t^2}{2} \end{cases}$$

At last, we have:

$$\begin{cases} v_n^{i+2} = v_n((i+1).\delta t+\tau) + a_n^{i+1}(\delta t-\tau) \\ v_{n-1}^{i+2} = v_{n-1}((i+1).\delta t+\tau) + a_{n-1}^{i+1}(\delta t-\tau) \\ d_n^{i+2} = d_n((i+1).\delta t+\tau) + (v_{n-1}((i+1).\delta t+\tau) - v_n((i+1).\delta t+\tau))(\delta t-\tau) \\ \qquad + (a_{n-1}^{i+1} - a_n^{i+1})\dfrac{(\delta t-\tau)^2}{2} \end{cases}$$

This system develops into:

$$\begin{cases} v_n^{i+2} = v_n^i + a_n^{i-1}\tau + a_n^i\delta t + a_n^{i+1}(\delta t-\tau) \\ v_{n-1}^{i+2} = v_{n-1}^i + a_{n-1}^{i-1}\tau + a_{n-1}^i\delta t + a_{n-1}^{i+1}(\delta t-\tau) \\ d_n^{i+2} = d_n^i + (v_{n-1}^i - v_n^i)2\delta t + (a_{n-1}^{i-1} - a_n^{i-1})\tau\left(2\delta t - \dfrac{\tau}{2}\right) \\ \qquad + (a_{n-1}^i - a_n^i)\delta t\left(\dfrac{3}{2}\delta t - \tau\right) + (a_{n-1}^{i+1} - a_n^{i+1})\dfrac{(\delta t-\tau)^2}{2} \end{cases}$$

In this system, we only keep a_n^i, bounding the other accelerations by either $a_{\min}$ or $a_{\max}$. We also remark that $\tau(\delta t - \tau/2) + (\delta t - \tau)^2/2 = \delta t^2/2$, $0 < 3\delta t/2 - \tau \leqslant 3\delta t/2$ and $\delta t\tau \leqslant \delta t^2$. Thus:

$$\begin{aligned}
&d_n^i + (v_{n-1}^i - v_n^i)2\delta t + (a_{n-1}^{i-1} - a_n^{i-1})\tau\left(2\delta t - \frac{\tau}{2}\right) \\
&\quad + (a_{n-1}^i - a_n^i)\delta t\left(\frac{3}{2}\delta t - \tau\right) + (a_{n-1}^{i+1} - a_n^{i+1})\frac{(\delta t - \tau)^2}{2} \\
&\quad \geqslant d_n^i + 2(v_{n-1}^i - v_n^i)\delta t + (a_{\min} - a_{\max})\left(\frac{\delta t^2}{2} + \delta t\tau\right) + \frac{3}{2}(a_{\min} - a_n^i)\delta t^2 \\
&\quad \geqslant \tilde{d}_n^i + (v_{n-1}^i - v_n^i)\delta t + (a_{\min} - a_{\max})\delta t\tau + \frac{3}{2}(a_{\min} - a_n^i)\delta t^2 \\
&\quad \geqslant \tilde{d}_n^i + (\tilde{v}_{n-1,n}^i - \tilde{v}_n^i)\delta t - \frac{3}{2}(a_n^i - a_{\min})\delta t^2
\end{aligned}$$

Finally, we find the following bounds:

$$\begin{cases}
v_n^{i+2} \leqslant \tilde{v}_n^i + a_n^i\delta t \\
v_{n-1}^{i+2} \geqslant \tilde{v}_{n-1,n}^i + a_{\min}\delta t \\
d_n^{i+2} \geqslant \tilde{d}_n^i + (\tilde{v}_{n-1,n}^i - \tilde{v}_n^i)\delta t - \frac{3}{2}(a_n^i - a_{\min})\delta t^2
\end{cases}$$

As a consequence, we have:

$$\begin{aligned}
\delta d_n^{i+2} \geqslant\ & \tilde{d}_n^i - d_{\text{crit}} + (\tilde{v}_{n-1,n}^i - \tilde{v}_n^i)\delta t - \frac{3}{2}(a_n^i - a_{\min})\delta t^2 \\
& + \min\left(0, \frac{(\tilde{v}_n^i + a_n^i\delta t)^2 - (\tilde{v}_{n-1,n}^i + a_{\min}\delta t)^2}{2a_{\min}}\right)
\end{aligned}$$

Thus, to ensure $\delta d_n^{i+2} \geqslant 0$, both following conditions are required:

$$\begin{cases}
\tilde{d}_n^i - d_{\text{crit}} + (\tilde{v}_{n-1,n}^i - \tilde{v}_n^i)\delta t - \frac{3}{2}(a_n^i - a_{\min})\delta t^2 & \geqslant 0 \\
\tilde{d}_n^i - d_{\text{crit}} + (\tilde{v}_{n-1,n}^i - \tilde{v}_n^i)\delta t - \frac{3}{2}(a_n^i - a_{\min})\delta t^2 + \frac{(\tilde{v}_n^i + a_n^i\delta t)^2 - (\tilde{v}_{n-1,n}^i + a_{\min}\delta t)^2}{2a_{\min}} & \geqslant 0
\end{cases}$$

The first condition can be rewritten as $a_n^i \leqslant a_{\min} + 2(\tilde{d}_n^i - d_{\text{crit}} + (\tilde{v}_{n-1,n}^i - \tilde{v}_n^i)\delta t)/(3\delta t^2)$, leading to the first argument of the $\min$ function of (2). As $\widetilde{\delta d}_n^i \geqslant 0$ (*cf.* note at the end of appendix I), this value is always correct (*i.e.* $\geqslant a_{\min}$).

The second condition develops into:

$$\begin{aligned}
&\tilde{d}_n^i - d_{\text{crit}} + \frac{{\tilde{v}_n^i}^2 - {\tilde{v}_{n-1,n}^i}^2}{2a_{\min}} + (\tilde{v}_{n-1,n}^i - \tilde{v}_n^i)\delta t \\
&\quad + \frac{3}{2}(a_{\min} - a_n^i)\delta t^2 + \frac{({a_n^i}^2 - {a_{\min}}^2)\delta t^2 + 2(\tilde{v}_n^i a_n^i - \tilde{v}_{n-1,n}^i a_{\min})\delta t}{2a_{\min}} \quad \geqslant 0
\end{aligned}$$

As $\widetilde{\delta d}_n^i = \tilde{d}_n^i - d_{\text{crit}} + \left(\tilde{v}_n^{i\,2} - \tilde{v}_{n-1,n}^{i\;\;2}\right)/(2a_{\text{min}})$, this can be simplified into:

$$\begin{aligned}
\widetilde{\delta d}_n^i + \frac{(a_n^{i\,2} - 3a_n^i a_{\text{min}} + 2a_{\text{min}}{}^2)\delta t^2 + 2\tilde{v}_n^i(a_n^i - a_{\text{min}})}{2a_{\text{min}}} &\geqslant 0 \\
\widetilde{\delta d}_n^i + \left(\left(\frac{a_n^i}{2} - a_{\text{min}}\right)\delta t + \tilde{v}_n^i\right)\frac{a_n^i - a_{\text{min}}}{a_{\text{min}}}\delta t &\geqslant 0
\end{aligned}$$

This last condition contains, once again, a quadratic function of a_n^i whose main (second degree) factor is negative ($\delta t^2/(2a_{\text{min}})$: the quadratic function is thus positive between its roots. As $\widetilde{\delta d}_n^i$, the value of this function for a_{min}, is positive, the lower root is smaller than a_{min}. This function can be developed into:

$$\begin{aligned}
&\widetilde{\delta d}_n^i + \frac{((a_n^i - 2a_{\text{min}})\delta t + 2\tilde{v}_n^i)(a_n^i - a_{\text{min}})\delta t}{2a_{\text{min}}} \\
&= \widetilde{\delta d}_n^i + \frac{\delta t a_n^{i\,2} + (2\tilde{v}_n^i - 3a_{\text{min}}\delta t)a_n^i - 2a_{\text{min}}(\tilde{v}_n^i - a_{\text{min}}\delta t)}{2a_{\text{min}}}\delta t \\
&= \frac{\delta t^2 . a_n^{i\,2} + (2\tilde{v}_n^i - 3a_{\text{min}}\delta t)\delta t . a_n^i + 2a_{\text{min}}(\widetilde{\delta d}_n^i - (\tilde{v}_n^i - a_{\text{min}}\delta t)\delta t)}{2a_{\text{min}}}
\end{aligned}$$

The discriminant of the numerator of this function is:

$$\begin{aligned}
&(2\tilde{v}_n^i - 3a_{\text{min}}\delta t)^2\delta t^2 - 8\delta t^2 a_{\text{min}}(\widetilde{\delta d}_n^i - (\tilde{v}_n^i - a_{\text{min}}\delta t)\delta t) \\
&= \delta t^2(4\tilde{v}_n^{i\,2} - 12a_{\text{min}}\delta t\tilde{v}_n^i + 9a_{\text{min}}{}^2\delta t^2 - 8\widetilde{\delta d}_n^i a_{\text{min}} + 8\tilde{v}_n^i a_{\text{min}}\delta t - 8a_{\text{min}}{}^2\delta t^2) \\
&= \delta t^2\left((2\tilde{v}_n^i - a_{\text{min}}\delta t)^2 - 8\widetilde{\delta d}_n^i a_{\text{min}}\right)
\end{aligned}$$

The upper root is thus:

$$\frac{\sqrt{\left(\tilde{v}_n^i - a_{\text{min}}\frac{\delta t}{2}\right)^2 - 2\widetilde{\delta d}_n^i a_{\text{min}}} - \left(\tilde{v}_n^i - \frac{3}{2}a_{\text{min}}\delta t\right)}{\delta t}$$

which is the second argument of the $\min$ function of (2).

We thus proved that $\widetilde{\delta d}_n^i \geqslant 0$ and $a_{\text{min}} \leqslant a_n^i \leqslant a_{\text{lim}}(\,d_n^i, v_n^i, v_{n-1}^i)$ implies $\delta d_n^{i+2} \geqslant 0$. We still have to prove this implies $\widetilde{\delta d}_n^{i+1} \geqslant 0$. As $\widetilde{\delta d}_n^{i+1} = \tilde{d}_n^{i+1} - d_{\text{crit}} + \left(\tilde{v}_{n-1,n}^{i+1\;2} - (\tilde{v}_n^{i+1})^2\right)/(-2a_{\text{min}})$, we need to find expressions for $\tilde{d}_n^{i+1}$, $\tilde{v}_{n-1,n}^{i+1}$ and $\tilde{v}_n^{i+1}$. We have:

$$\begin{cases}
\tilde{v}_n^{i+1} = v_n^{i+1} + a_{\text{max}}\delta t \\
\tilde{v}_{n-1,n}^{i+1} = v_{n-1}^{i+1} + a_{\text{min}}\delta t \\
\tilde{d}_n^{i+1} = d_n^{i+1} + (v_{n-1}^{i+1} - v_n^{i+1})\delta t + (a_{\text{min}} - a_{\text{max}})\delta t^2/2
\end{cases}$$

and

$$\begin{cases} v_n^{i+1} = v_n(i.\delta t + \tau) + a_n^i(\delta t - \tau) \\ v_{n-1}^{i+1} = v_{n-1}(i.\delta t + \tau) + a_{n-1}^i(\delta t - \tau) \\ d_n^{i+1} = d_n(i.\delta t + \tau) + (v_{n-1}(i.\delta t + \tau) - v_n(i.\delta t + \tau))(\delta t - \tau) + (a_{n-1}^i - a_n^i)\dfrac{(\delta t - \tau)^2}{2} \end{cases}$$

where we already saw that

$$\begin{cases} v_n(i.\delta t + \tau) = v_n^i + a_n^{i-1}\tau \\ v_{n-1}(i.\delta t + \tau) = v_{n-1}^i + a_{n-1}^{i-1}\tau \\ d_n(i.\delta t + \tau) = d_n^i + (v_{n-1}^i - v_n^i)\tau + (a_{n-1}^{i-1} - a_n^{i-1})\dfrac{\tau^2}{2} \end{cases}$$

Thus, we have:

$$\begin{cases} v_n^{i+1} = v_n^i + a_n^{i-1}\tau + a_n^i(\delta t - \tau) \\ v_{n-1}^{i+1} = v_{n-1}^i + a_{n-1}^{i-1}\tau + a_{n-1}^i(\delta t - \tau) \\ d_n^{i+1} = d_n^i + (v_{n-1}^i - v_n^i)\tau + (a_{n-1}^{i-1} - a_n^{i-1})\dfrac{\tau^2}{2} + (v_{n-1}^i - v_n^i)(\delta t - \tau) \\ \qquad + (a_{n-1}^{i-1} - a_n^{i-1})\tau(\delta t - \tau) + (a_{n-1}^i - a_n^i)\dfrac{(\delta t - \tau)^2}{2} \\ \quad = d_n^i + (v_{n-1}^i - v_n^i)\delta t + (a_{n-1}^{i-1} - a_n^{i-1})\tau\left(\delta t - \dfrac{\tau}{2}\right) + (a_{n-1}^i - a_n^i)\dfrac{(\delta t - \tau)^2}{2} \end{cases}$$

and

$$\begin{cases} \tilde{v}_n^{i+1} = v_n^i + a_n^{i-1}\tau + a_n^i(\delta t - \tau) + a_{\text{max}}\delta t \\ \tilde{v}_{n-1,n}^{i+1} = v_{n-1}^i + a_{n-1}^{i-1}\tau + a_{n-1}^i(\delta t - \tau) + a_{\text{min}}\delta t \\ \tilde{d}_n^{i+1} = d_n^i + 2(v_{n-1}^i - v_n^i)\delta t + (a_{n-1}^{i-1} - a_n^{i-1})\tau\left(2\delta t - \dfrac{\tau}{2}\right) \\ \qquad + (a_{n-1}^i - a_n^i)\dfrac{(\delta t - \tau)(3\delta t - \tau)}{2} + (a_{\text{min}} - a_{\text{max}})\dfrac{\delta t^2}{2} \\ \quad = \tilde{d}_n^i + (v_{n-1}^i - v_n^i)\delta t + (a_{n-1}^{i-1} - a_n^{i-1})\tau\left(2\delta t - \dfrac{\tau}{2}\right) + (a_{n-1}^i - a_n^i)\dfrac{(\delta t - \tau)(3\delta t - \tau)}{2} \end{cases}$$

Finally, as $\tau(2\delta t - \tau/2) + (\delta t - \tau)(3\delta t - \tau)/2 = 3\delta t^2/2$:

$$\begin{cases} \tilde{v}_n^{i+1} \leqslant \tilde{v}_n^i + \max\left(a_n^{i-1}, a_n^i\right)\delta t \\ \tilde{v}_{n-1,n}^{i+1} \geqslant \tilde{v}_{n-1,n}^i + a_{\text{min}}\delta t \\ \tilde{d}_n^{i+1} \geqslant \tilde{d}_n^i + (\tilde{v}_{n-1,n}^i - \tilde{v}_n^i)\delta t + (a_{\text{max}} - a_{\text{min}})\delta t^2 - \dfrac{3}{2}\left(\max\left(a_n^{i-1}, a_n^i\right) - a_{\text{min}}\right)\delta t^2 \end{cases} \tag{4}$$

$\widetilde{\delta d}_n^{i+1}$ can thus be lower bounded by:

$$
\begin{aligned}
&\tilde{d}_n^{i+1} - d_{\text{crit}} + \frac{{\tilde{v}_{n-1,n}^{i+1}}^2 - (\tilde{v}_n^{i+1})^2}{-2a_{\text{min}}} \\
&\geqslant \tilde{d}_n^i + (\tilde{v}_{n-1,n}^i - \tilde{v}_n^i)\delta t + (a_{\text{max}} - a_{\text{min}})\delta t^2 - \frac{3}{2}\left(\max\left(a_n^{i-1}, a_n^i\right) - a_{\text{min}}\right)\delta t^2 - d_{\text{crit}} \\
&\quad + \frac{(\tilde{v}_{n-1,n}^i + a_{\text{min}}\delta t)^2 - (\tilde{v}_n^i + \max\left(a_n^{i-1}, a_n^i\right)\delta t)^2}{-2a_{\text{min}}} \\
&\geqslant \tilde{d}_n^i - d_{\text{crit}} + \frac{{\tilde{v}_{n-1,n}^i}^2 - {\tilde{v}_n^i}^2}{-2a_{\text{min}}} + (a_{\text{max}} - a_{\text{min}})\delta t^2 - \frac{3}{2}\left(\max\left(a_n^{i-1}, a_n^i\right) - a_{\text{min}}\right)\delta t^2 \\
&\quad + (\tilde{v}_{n-1,n}^i - \tilde{v}_n^i)\delta t + \frac{\left({a_{\text{min}}}^2 - \max\left(a_n^{i-1}, a_n^i\right)^2\right)\delta t + 2\left(\tilde{v}_{n-1,n}^i a_{\text{min}} - \tilde{v}_n^i \max\left(a_n^{i-1}, a_n^i\right)\right)}{-2a_{\text{min}}}\delta t \\
&\geqslant \widetilde{\delta d}_n^i + (a_{\text{max}} - a_{\text{min}})\delta t^2 - \frac{3}{2}\left(\max\left(a_n^{i-1}, a_n^i\right) - a_{\text{min}}\right)\delta t^2 \\
&\quad + \frac{\left({a_{\text{min}}}^2 - \max\left(a_n^{i-1}, a_n^i\right)^2\right)\delta t + 2\left(a_{\text{min}} - \max\left(a_n^{i-1}, a_n^i\right)\right)\tilde{v}_n^i}{-2a_{\text{min}}}\delta t \\
&\geqslant \widetilde{\delta d}_n^i + (a_{\text{max}} - a_{\text{min}})\delta t^2 - \frac{\max\left(a_n^{i-1}, a_n^i\right) - a_{\text{min}}}{-a_{\text{min}}}\left(\tilde{v}_n^i + \left(\frac{1}{2}\max\left(a_n^{i-1}, a_n^i\right) - a_{\text{min}}\right)\delta t\right)\delta t
\end{aligned}
$$

This lower bound of $\widetilde{\delta d}_n^{i+1}$ is similar to the one found previously for δd_n^{i+2}. The two differences are:

- a_n^i is replaced by $\max\left(a_n^{i-1}, a_n^i\right)$; as a consequence, $\widetilde{\delta d}_n^{i+1} \geqslant 0$ leads to conditions similar to those found for $\delta d_n^{i+2} \geqslant 0$, but they have to be respected by a_n^i and a_n^{i-1};
- the lower bound on $\tilde{d}_n^{i+1}$ is higher than the one found for d_n^{i+2}; conditions on a_n^i implying $\delta d_n^{i+2} \geqslant 0$ thus insure that those on a_n^i for $\widetilde{\delta d}_n^{i+1} \geqslant 0$ are respected.

The only condition to ensure $\widetilde{\delta d}_n^{i+1} \geqslant 0$ is thus, when $i > 0$ and $a_n^{i-1} > a_n^i$:

$$\widetilde{\delta d}_n^i + (a_{\text{max}} - a_{\text{min}})\delta t^2 - \frac{a_n^{i-1} - a_{\text{min}}}{-a_{\text{min}}}\left(\tilde{v}_n^i + \left(\frac{a_n^{i-1}}{2} - a_{\text{min}}\right)\delta t\right)\delta t \quad \geqslant 0 \tag{5}$$

When $i = 0$, this implies (as $a_n^{-1} = 0$):

$$\widetilde{\delta d}_n^0 + (a_{\text{max}} - a_{\text{min}})\delta t^2 - \left(\tilde{v}_n^0 - a_{\text{min}}\delta t\right)\delta t \geqslant 0$$

This is true as:

$$\widetilde{\delta d}_n^0 + (a_{\text{max}} - a_{\text{min}})\delta t^2 - \left(\tilde{v}_n^0 - a_{\text{min}}\delta t\right)\delta t \quad = \widetilde{\delta d}_n^0 - \tilde{v}_n^0\delta t + a_{\text{max}}\delta t^2 = \widetilde{\delta d}_n^0 - v_n^0\delta t$$

When $i > 0$, $\widetilde{\delta d}_n^i$ and $\tilde{v}_n^i$ are not known when a_n^{i-1} is computed (at time $(i-1)\delta t$), as they depend of the velocities and distance perceived at time $i\delta t$. We thus need to bound them, using velocities and distance perceived at time $(i-1)\delta t$ and a simple formula derived from system (4), where $\max(...)$ is replaced by a_{max}:

$$\begin{cases} \tilde{v}_n^i \leqslant \tilde{v}_n^{i-1} + a_{\text{max}}\delta t \\ \widetilde{\delta d}_n^i \geqslant \widetilde{\delta d}_n^{i-1} + (a_{\text{max}} - a_{\text{min}})\delta t^2 - \dfrac{a_{\text{max}} - a_{\text{min}}}{-a_{\text{min}}} \left(\tilde{v}_n^{i-1} + \left(\dfrac{a_{\text{max}}}{2} - a_{\text{min}} \right) \delta t \right) \delta t \\ \qquad \geqslant \widetilde{\delta d}_n^{i-1} - \dfrac{a_{\text{max}} - a_{\text{min}}}{-a_{\text{min}}} \left(\tilde{v}_n^{i-1} + \dfrac{a_{\text{max}}}{2}\delta t \right) \delta t \end{cases}$$

Second line can be improved, as $\widetilde{\delta d}_n^i$ is already known to be positive. Thus, inequality (5) becomes when $i > 0$:

$$\begin{aligned} &\max\left(0, \widetilde{\delta d}_n^{i-1} - \frac{a_{\text{max}} - a_{\text{min}}}{-a_{\text{min}}} \left(\tilde{v}_n^{i-1} + \frac{a_{\text{max}}}{2}\delta t \right) \delta t \right) + (a_{\text{max}} - a_{\text{min}})\delta t^2 \\ &- \frac{a_n^{i-1} - a_{\text{min}}}{-a_{\text{min}}} \left(\tilde{v}_n^{i-1} + \left(\frac{a_n^{i-1}}{2} + a_{\text{max}} - a_{\text{min}} \right) \delta t \right) \delta t \qquad \geqslant 0 \end{aligned}$$

Once again, left part of this inequality is a second degree polynomial of a_n^{i-1}, with a negative second degree factor and a positive value $\widetilde{D}_n^{i-1}$ when $a_n^{i-1} = a_{\text{min}}$. This polynomial develops into:

$$\begin{aligned} &\widetilde{D}_n^{i-1} - \frac{\left(a_n^{i-1} - a_{\text{min}}\right)\left(2\tilde{v}_n^{i-1} + \left(a_n^{i-1} + 2(a_{\text{max}} - a_{\text{min}})\right)\delta t\right)}{-2a_{\text{min}}}\delta t \\ &= -\frac{\delta t^2 \left(a_n^{i-1}\right)^2 + \left(2\tilde{v}_n^{i-1} + (2a_{\text{max}} - 3a_{\text{min}})\delta t\right)\delta t a_n^{i-1}}{-2a_{\text{min}}} \\ &\quad + \left(\widetilde{D}_n^{i-1} - \left(\tilde{v}_n^{i-1} + (a_{\text{max}} - a_{\text{min}})\delta t\right)\delta t\right) \end{aligned}$$

The discriminant of this polynomial numerator (*i.e.* $-2a_{\text{min}}$ times this polynomial) is:

$$\begin{aligned} &\left(2\tilde{v}_n^{i-1} + (2a_{\text{max}} - 3a_{\text{min}})\delta t\right)^2 \delta t^2 - 4\delta t^2 2a_{\text{min}} \left(\widetilde{D}_n^{i-1} - \left(\tilde{v}_n^{i-1} + (a_{\text{max}} - a_{\text{min}})\delta t\right)\delta t\right) \\ &= 4\delta t^2 \left(\left(\tilde{v}_n^{i-1} + \left(a_{\text{max}} - \frac{3}{2}a_{\text{min}} \right) \delta t \right)^2 - 2a_{\text{min}} \left(\widetilde{D}_n^{i-1} - \left(\tilde{v}_n^{i-1} + (a_{\text{max}} - a_{\text{min}})\delta t \right) \delta t \right) \right) \\ &= 4\delta t^2 \left(\left(\tilde{v}_n^{i-1}\right)^2 + \left({a_{\text{max}}}^2 + \frac{9}{4}{a_{\text{min}}}^2 - 3a_{\text{max}}a_{\text{min}} \right) \delta t^2 + 2\tilde{v}_n^{i-1}\delta t \left(a_{\text{max}} - \frac{3}{2}a_{\text{min}} \right) \right. \\ &\qquad \left. -2a_{\text{min}}\widetilde{D}_n^{i-1} + 2a_{\text{min}}\tilde{v}_n^{i-1}\delta t + 2a_{\text{min}}a_{\text{max}}\delta t^2 - 2{a_{\text{min}}}^2\delta t^2 \right) \\ &= 4\delta t^2 \left(\left(\tilde{v}_n^{i-1}\right)^2 + \left({a_{\text{max}}}^2 + \frac{{a_{\text{min}}}^2}{4} - a_{\text{max}}a_{\text{min}} \right) \delta t^2 + 2\tilde{v}_n^{i-1}\delta t \left(a_{\text{max}} - \frac{a_{\text{min}}}{2} \right) - 2a_{\text{min}}\widetilde{D}_n^{i-1} \right) \\ &= 4\delta t^2 \left(\left(\tilde{v}_n^{i-1} + \left(a_{\text{max}} - \frac{a_{\text{min}}}{2} \right) \delta t \right)^2 - 2a_{\text{min}}\widetilde{D}_n^{i-1} \right) \end{aligned}$$

As a conclusion, the only condition for inequality (5) to be respected is that a_n^{i-1} remains lower than the upper root, which is:

$$\frac{\sqrt{\left(\tilde{v}_n^{i-1} + \left(a_{\max} - \frac{a_{\min}}{2}\right)\delta t\right)^2 - 2a_{\min}\widetilde{D}_n^{i-1}} - \left(\tilde{v}_n^{i-1} + \left(a_{\max} - \frac{3}{2}a_{\min}\right)\delta t\right)}{\delta t}$$

This is insured by the third term of $a_{\lim}(d_n^{i-1}, v_n^{i-1}, v_{n-1}^{i-1})$.

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